

Lepton masses and mixing in $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ models with a S_3 flavor symmetry

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We propose a model based on the gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ with an extra S_3 flavor symmetry, which accounts for the lepton masses and mixing. The small active neutrino masses are generated via a double seesaw mechanism. In this scenario, the spectrum of neutrinos presents very light, light and very heavy masses. The model predicts a quasidegenerate normal hierarchy active neutrino mass spectrum and the relation $\Delta m_{21}^2 \ll \Delta m_{31}^2$ arises from effective six-dimensional operators. The obtained neutrino mixing parameters are in agreement with the neutrino oscillation experimental data. We find CP violation in neutrino oscillations with a Jarlskog invariant of about 10^{-2} .

I. INTRODUCTION

The experimental confirmation of the electroweak symmetry breaking (EWSB) sector of the Standard Model (SM) given by the discovery of the Higgs Boson at the LHC [1, 2] has concentered its great success in describing electroweak phenomena. However, the SM does not explain neither the pattern of fermion masses and mixing nor the existence of three generations of fermions. In consequence, to address these issues it is necessary to consider a more fundamental theory. The existing pattern of fermion masses goes over a range of five orders of magnitude in the quark sector and a much wider range when neutrinos are included. While in the quark sector the mixing angles are small, in the lepton sector two of the mixing angles are large, and one is small; this suggests that the corresponding mechanisms for masses and mixings should be different. Experiments with solar, atmospheric and reactor neutrinos [3–8] have brought evidence of neutrino oscillations caused by nonzero mass. The global fits of the available data from the Daya Bay [4], T2K [5], MINOS [6], Double CHOOZ [7] and RENO [8] neutrino oscillation experiments, constrain the neutrino mass squared splittings and mixing parameters [9].

Models with an extended gauge symmetry are frequently used to tackle the limitations of the SM. In particular, those based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry, called 331 for short, can explain the origin of fermion generations thanks to the introduction of a family nonuniversal $U(1)_X$ symmetry [10, 11]. Specific realizations of 331 models have several appealing features. First, the three family structure in the fermion sector is a consequence of the chiral anomaly cancellation [12] and the asymptotic freedom in QCD. Second, the large mass splitting between the heaviest quark family and the two lighter ones can be explained since the former is in a different $U(1)_X$ representation [13]. Third, these models include a natural Peccei-Quinn symmetry, that sheds light on the strong-CP problem [14]. Finally, versions with heavy sterile neutrinos have cold dark matter candidates as weakly interacting massive particles (WIMPs) [15]. We consider 331 models with a scalar sector composed of three $SU(3)_L$ scalar triplets, where one heavy triplet field acquires a vacuum expectation value (VEV) at a high energy scale, v_χ , responsible for breaking the symmetry $SU(3)_L \otimes U(1)_X$ down to the SM electroweak gauge group $SU(2)_L \otimes U(1)_Y$; and two lighter triplets get VEVs v_ρ and v_η at the electroweak scale, thus triggering the EWSB.

On the other hand, discrete flavor symmetries are important ingredients in models of particle masses and mixing, and many of them have been considered to resolve the fermion mass hierarchy; for recent reviews see Refs. [16–19]. In particular the S_3 flavor symmetry is a very good candidate for explaining the prevailing pattern of fermion masses and mixing. The S_3 discrete symmetry is the smallest non-Abelian discrete symmetry group having three irreducible representations (irreps), explicitly two singlets and one doublet irreps [20]. Since two of the three $SU(3)_L$ scalar triplets of the 331 models belong to the same $U(1)_X$ representation while the third is in a different one, the scalar fields can be arranged into doublet and non trivial S_3 singlet irreps. Regarding charged leptons, we accommodate left-

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and right-handed leptons as well as one heavy Majorana neutrino into S_3 singlet representations, and the remaining two heavy Majorana neutrinos into a S_3 doublet representation. We assume that the heavy Majorana neutrinos have masses much larger than the TeV scale, so that the hierarchy $M_R \gg v_\chi \gg v_\rho, v_\eta$ is fulfilled, implying that the small active neutrino masses are generated via a double seesaw mechanism. This mechanism does not include any exotic charges, neither in the fermionic nor in the scalar sector [21]. We predict a quasidegenerate normal hierarchy active neutrino mass spectrum and the relation $\Delta m_{21}^2 \ll \Delta m_{31}^2$ results from effective six-dimensional Yukawa terms.

This paper is organized as follows. In Sec. II we explain some theoretical aspects of the 331 model with $\beta = -\frac{1}{\sqrt{3}}$ and its particle content, as well as the particle assignments under doublet and singlet S_3 representations, in particular in the fermionic and scalar sector. In Sec. III we focus on the discussion of neutrino masses and mixing and give our corresponding results. Conclusions are given in Sec. IV. In the appendices we present several technical details: Appendix A gives a brief description of the S_3 group; Appendix B shows the diagonalization of the neutrino mass matrix.

II. A $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes S_3$ MODEL WITH $\beta = -\frac{1}{\sqrt{3}}$

A. Particle content

We consider a 331 model with $\beta = -\frac{1}{\sqrt{3}}$ [21–23], where the electric charge is defined in terms of $SU(3)$ generators and the identity by

$$Q = T_3 - \frac{1}{\sqrt{3}}T_8 + XI, \quad (1)$$

with $I = \text{Diag}(1, 1, 1)$, $T_3 = \frac{1}{2}\text{Diag}(1, -1, 0)$ and $T_8 = (\frac{1}{2\sqrt{3}})\text{Diag}(1, 1, -2)$. To avoid chiral anomalies, fermions are assigned to the following $(SU(3)_C, SU(3)_L, U(1)_X)$ left- and right-handed representations:

$$\begin{aligned} Q_L^{1,2} &= \begin{pmatrix} D^{1,2} \\ -U^{1,2} \\ J^{1,2} \end{pmatrix}_L : (3, 3^*, 0), & \begin{cases} D_R^{1,2} : (3^*, 1, -1/3) \\ U_R^{1,2} : (3^*, 1, 2/3) \\ J_R^{1,2} : (3^*, 1, -1/3) \end{cases}, \\ Q_L^3 &= \begin{pmatrix} U^3 \\ D^3 \\ T \end{pmatrix}_L : (3, 3, 1/3), & \begin{cases} U_R^3 : (3^*, 1, 2/3) \\ D_R^3 : (3^*, 1, -1/3) \\ T_R : (3^*, 1, 2/3) \end{cases}, \\ L_L^{1,2,3} &= \begin{pmatrix} \nu^{1,2,3} \\ e^{1,2,3} \\ (\nu^{1,2,3})^c \end{pmatrix}_L : (1, 3, -1/3), & \begin{cases} e_R^{1,2,3} : (1, 1, -1) \\ N_R^{1,2,3} : (1, 1, 0) \end{cases}, \end{aligned} \quad (2)$$

where U_L^i and D_L^i for $i = 1, 2, 3$ are three up- and down-type quark components in the flavor basis, while ν_L^i and e_L^i are the neutral and charged leptons. The right-handed components transform as singlets under $SU(3)_L$ with $U(1)_X$ quantum numbers corresponding to the electric charges.

Additionally, the model includes heavy fermions with the following properties: a single flavor quark T with electric charge $2/3$, two flavor quarks $J^{2,3}$ with charge $-1/3$, three neutral Majorana leptons $(\nu^{1,2,3})_L^c$ and three right-handed Majorana leptons $N_R^{1,2,3}$.

The scalar sector consists of a triplet field χ , which provides the masses to the new heavy fermions, and two triplets ρ and η , which give masses to the SM fermions at the electroweak scale. The $(SU(3)_L, U(1)_X)$ group structure of the

scalar fields is:

$$\begin{aligned}
\chi &= \begin{pmatrix} \chi_1^0 + \frac{1}{\sqrt{2}} w_\chi e^{i\varphi_\chi} \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(v_\chi + \xi_\chi \pm i\zeta_\chi) \end{pmatrix} : (3, -1/3) \\
\rho &= \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v_\rho + \xi_\rho \pm i\zeta_\rho) \\ \rho_3^+ \end{pmatrix} : (3, 2/3) \\
\eta &= \begin{pmatrix} \frac{1}{\sqrt{2}}(v_\eta + \xi_\eta \pm i\zeta_\eta) \\ \eta_2^- \\ \eta_3^0 + \frac{1}{\sqrt{2}} w_\eta e^{i\varphi_\eta} \end{pmatrix} : (3, -1/3).
\end{aligned} \tag{3}$$

The electroweak symmetry breaking (EWSB) mechanism follows

$$SU(3)_L \otimes U(1)_X \xrightarrow{\langle \chi \rangle} SU(2)_L \otimes U(1)_Y \xrightarrow{\langle \eta \rangle, \langle \rho \rangle} U(1)_Q,$$

where the vacuum expectation values satisfy the hierarchy $v_\chi \gg v_\eta, v_\rho \gg w_\chi, w_\eta$. Notice that we have introduced nonvanishing complex vacuum expectation values in the first and third components of the χ and η triplets, respectively, as done in Refs. [24, 25].

In order to reduce the number of parameters in the Yukawa and scalar sectors of the 331 Lagrangian, we impose a S_3 flavor symmetry for fermions and scalars, making $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes S_3$ the full symmetry of our model. Apart from easily accommodating maximal mixing through its doublet representation, the S_3 discrete group has two different singlet representations crucial for reproducing the fermion masses [20]. The scalar fields are assigned into doublet and singlet representations of S_3 as follows,

$$\Phi = (\eta, \chi) \sim \mathbf{2}, \quad \rho \sim \mathbf{1}', \tag{4}$$

whereas the leptons transform under S_3 as

$$L_L^{1,2,3} \sim \mathbf{1}, \quad e_R^{1,2,3} \sim \mathbf{1}', \quad N_R^1 \sim \mathbf{1}, \quad N_R = (N_R^2, N_R^3) \sim \mathbf{2}. \tag{5}$$

The corresponding S_3 assignments for quarks as well as the quark masses and mixing are studied in detail in the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes S_3$ model of Ref. [23].

With the above spectrum, we obtain the following Yukawa terms for the lepton sector invariant under S_3 :

$$\begin{aligned}
-\mathcal{L}_Y^{(L)} &= h_{\rho e}^{(L)} \bar{L}_L \rho e_R + h_\Phi^{(L)} \bar{L}_L (\Phi N_R)_1 + \frac{1}{2} m_N^{(1)} \bar{N}_R^1 N_R^{1C} + \frac{1}{2} m_N (\bar{N}_R N_R^C)_1 + \frac{h_\Phi^{(N)}}{\Lambda} \bar{N}_R^1 N_R^C (\Phi \Phi^\dagger)_2 \\
&\quad + \frac{h_\rho}{\Lambda^2} \bar{L}_L^a (L_L^C)^b \rho^c \varepsilon_{abc} (\Phi^\dagger \Phi)_{1'} + \text{H.c.}
\end{aligned} \tag{6}$$

Note that the heavy Majorana neutrinos N_R^2 and N_R^3 belonging to the same S_3 doublet have the same mass m_N , which is in general different than the mass $m_N^{(1)}$ of the heavy Majorana neutrino \bar{N}_R^1 . Therefore, the S_3 flavor symmetry leads to a heavy Majorana neutrino mass splitting, so that $m_N^{(1)} = \kappa m_N$ where the dimensionless parameter κ may differ from 1.

In order to see if there are operators of dimension larger than four that contribute to the neutrino masses, first we consider the bilinear combinations of two leptonic fields of Eq. (5). In the Yukawa terms given by Eq. (6), we have already found the combinations $\bar{L}_L N_R$, $\bar{N}_R^1 N_R^{1C}$, $\bar{N}_R N_R^C$, $\bar{L}_L L_L^C$ and $\bar{N}_R^1 N_R^C$; the only combination missing is $\bar{L}_L N_R^1$. Table I shows the $SU(3)_L \otimes S_3$ invariant operators with dimension larger than four built from these bilinears that could contribute to the neutrino masses. Only the terms $\frac{1}{\Lambda} \bar{N}_R^1 N_R^C (\Phi \Phi^\dagger)$ and $\frac{1}{\Lambda^2} \bar{L}_L L_L^C \rho (\Phi^\dagger \Phi)$ have vanishing $U(1)_X$ charge and thus they are invariant under the group $SU(3)_L \otimes U(1)_X \otimes S_3$. The five-dimensional Yukawa term gives a subleading contribution to the heavy Majorana neutrino masses. That contribution is suppressed by factor of about $\sim \frac{v_\Lambda^2}{\Lambda} \ll m_N$ where Λ is the cutoff of our model.

Operator	$SU(3)_L$	S_3	$U(1)_X$
$\frac{1}{\Lambda} \overline{L}_L N_R^1 (\Phi^\dagger \Phi^\dagger)$	Invariant	Invariant	$\neq 0$
$\frac{1}{\Lambda} \overline{N}_R^1 N_R^C (\Phi \rho^\dagger)$	Invariant	Invariant	$\neq 0$
$\frac{1}{\Lambda} \overline{N}_R^1 N_R^C (\Phi \Phi^\dagger)$	Invariant	Invariant	$= 0$
$\frac{1}{\Lambda^2} \overline{L}_L N_R^1 \rho (\Phi^\dagger \Phi)$	Invariant	Invariant	$\neq 0$
$\frac{1}{\Lambda^2} \overline{L}_L L_R^C \rho (\Phi^\dagger \Phi)$	Invariant	Invariant	$= 0$

Table I: $SU(3)_L \otimes S_3$ invariant operators with dimension larger than four that could contribute to the neutrino masses.

B. Scalar potential

The scalar potential of the model is constructed with the S_3 doublet $\Phi = (\eta, \chi)$ and the nontrivial S_3 singlet ρ fields, in the way invariant under the group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes S_3$. It is given by:

$$\begin{aligned}
V_H = & \mu_\rho^2 (\rho^\dagger \rho) + \mu_\Phi^2 (\Phi^\dagger \Phi)_1 + \lambda_1 (\rho^\dagger \rho) (\rho^\dagger \rho) + \lambda_2 (\Phi^\dagger \Phi)_1 (\Phi^\dagger \Phi)_1 + \lambda_3 (\Phi^\dagger \Phi)_1 (\Phi^\dagger \Phi)_1 \\
& + \lambda_4 (\Phi^\dagger \Phi)_2 (\Phi^\dagger \Phi)_2 + \lambda_5 (\rho^\dagger \rho) (\Phi^\dagger \Phi)_1 + \lambda_6 ((\rho^\dagger \Phi) (\Phi^\dagger \rho))_1 \\
& + f [\varepsilon^{ijk} (\Phi_i \Phi_j)_1 \rho_k + \text{H.c.}],
\end{aligned} \tag{7}$$

where $\Phi_i = (\eta_i, \chi_i)$ is a S_3 doublet with $i = 1, 2, 3$ and all parameters of the scalar potential have to be real.

We softly break the S_3 symmetry in the quadratic term of the scalar potential since the vacuum expectation values of the scalar fields η and χ contained in the S_3 doublet Φ satisfy the hierarchy $v_\chi \gg v_\eta$. Then, considering the quadratic S_3 soft-breaking terms $(\mu_\eta^2 - \mu_\chi^2) (\eta^\dagger \eta)$ and $\mu_{\eta\chi}^2 (\chi^\dagger \eta) + \text{H.c.}$ using the multiplication rules of the S_3 group, the scalar potential can be written in terms of the three scalar triplets as follows:

$$\begin{aligned}
V_H = & \mu_\rho^2 (\rho^\dagger \rho) + \mu_\eta^2 (\eta^\dagger \eta) + \mu_\chi^2 (\chi^\dagger \chi) + \mu_{\eta\chi}^2 [(\chi^\dagger \eta) + (\eta^\dagger \chi)] + \lambda_1 (\rho^\dagger \rho)^2 + (\lambda_2 + \lambda_4) [(\chi^\dagger \chi)^2 + (\eta^\dagger \eta)^2] \\
& + \lambda_5 [(\rho^\dagger \rho) (\chi^\dagger \chi) + (\rho^\dagger \rho) (\eta^\dagger \eta)] + 2(\lambda_2 - \lambda_4) (\chi^\dagger \chi) (\eta^\dagger \eta) + 2(\lambda_4 - \lambda_3) (\chi^\dagger \eta) (\eta^\dagger \chi) \\
& + \lambda_6 [(\chi^\dagger \rho) (\rho^\dagger \chi) + (\eta^\dagger \rho) (\rho^\dagger \eta)] + (\lambda_3 + \lambda_4) [(\chi^\dagger \eta)^2 + (\eta^\dagger \chi)^2] \\
& + 2f (\varepsilon^{ijk} \eta_i \chi_j \rho_k + \text{H.c.}).
\end{aligned} \tag{8}$$

It is worth mentioning that the S_3 soft-breaking term $\mu_{\eta\chi}^2 (\chi^\dagger \eta) + \text{H.c.}$ is not relevant for the minimization conditions of the scalar potential as well as for the masses of the physical scalars.

Considering $f, v_\chi \gg v_\eta, v_\rho$, we found in detail in Ref. [23] the physical scalar mass eigenstates. The CP-even scalar mass eigenstates are

$$\begin{pmatrix} H_1^0 \\ h^0 \end{pmatrix} \simeq \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \xi_\rho \\ \xi_\eta \end{pmatrix}, \quad H_3^0 \simeq \xi_\chi. \tag{9}$$

The CP-odd scalar mass eigenstates are

$$\begin{pmatrix} A^0 \\ G_1^0 \\ G_3^0 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta & 0 \\ \sin \beta & -\cos \beta & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \zeta_\rho \\ \zeta_\eta \\ \zeta_\chi \end{pmatrix}. \tag{10}$$

The charged scalar mass eigenstates are

$$\begin{pmatrix} H_1^\pm \\ G_1^\pm \end{pmatrix} = \begin{pmatrix} \cos \nu & \sin \nu \\ \sin \nu & -\cos \nu \end{pmatrix} \begin{pmatrix} \rho_1^\pm \\ \eta_2^\pm \end{pmatrix}, \quad \begin{pmatrix} H_2^\pm \\ G_2^\pm \end{pmatrix} = \begin{pmatrix} \cos \gamma & \sin \gamma \\ \sin \gamma & -\cos \gamma \end{pmatrix} \begin{pmatrix} \rho_3^\pm \\ \chi_2^\pm \end{pmatrix}. \quad (11)$$

The remaining neutral scalar mass eigenstates are

$$H_2^0 \simeq \eta_3^0, \quad \overline{H}_2^0 \simeq \overline{\eta}_3^0, \quad G_2^0 \simeq -\chi_1^0, \quad \overline{G}_2^0 \simeq -\overline{\chi}_1^0, \quad (12)$$

The mixing angles of the physical scalar fields are

$$\tan \alpha \simeq \tan \beta \simeq \tan \nu \simeq \frac{v_\rho}{v_\eta}, \quad \tan \gamma \simeq \frac{v_\rho}{v_\chi}. \quad (13)$$

Notice that after the spontaneous breaking of the gauge symmetry $SU(3)_L \otimes U(1)_X$ and rotations into mass eigenstates, the model contains four massive charged Higgs (H_1^\pm, H_2^\pm), one CP-odd Higgs (A^0), three neutral CP-even Higgs (h^0, H_1^0, H_3^0) and two neutral Higgs (H_2^0, \overline{H}_2^0) bosons. Here we identify the scalar h^0 with the SM-like 126 GeV Higgs boson observed at the LHC. We recall that the neutral Goldstone bosons G_1^0, G_3^0 correspond to the Z, Z' gauge bosons, respectively, while the remaining neutral Goldstone bosons G_2^0, \overline{G}_2^0 correspond to the K^0, \overline{K}^0 gauge bosons, respectively. Furthermore, the charged Goldstone bosons G_1^\pm and G_2^\pm correspond to the W^\pm and K^\pm gauge bosons, respectively [10, 11].

In Ref. [23] we follow the method described in Ref. [26] to show that the scalar potential is stable when its quartic couplings satisfy the following relations:

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_6 > 0, \quad \lambda_2 > \lambda_3, \quad \lambda_2 + \lambda_4 > 0, \quad \lambda_5 + \lambda_6 > 2\sqrt{\lambda_1(\lambda_2 + \lambda_4)}. \quad (14)$$

III. LEPTON MASSES AND MIXING

A. Neutrino masses

From Eq. (6), and using the product rules for the S_3 group given in Appendix A, it follows that the Yukawa mass terms for the lepton sector are given by

$$\begin{aligned} -\mathcal{L}_{mass}^{(L)} &= \frac{v_\rho}{\sqrt{2}} \overline{e}_L h_{\rho e}^{(L)} e_R + \frac{v_\eta}{\sqrt{2}} \overline{\nu}_L h_\Phi^{(L)} N_R^2 + \frac{v_\chi}{\sqrt{2}} \overline{\nu}_R^C h_\Phi^{(L)} N_R^3 \\ &+ \frac{1}{2} m_N^{(1)} \overline{N}_R^1 N_R^{1C} + \frac{1}{2} m_N \left(\overline{N}_R^2 N_R^{2C} + \overline{N}_R^3 N_R^{3C} \right) \\ &+ i \frac{(w_\chi v_\eta \sin \varphi_\chi - w_\eta v_\chi \sin \varphi_\eta) v_\rho}{\sqrt{2} \Lambda^2} (\overline{\nu}_R^C h_\rho \nu_L^C - \overline{\nu}_L h_\rho \nu_R) + \text{H.c.} \end{aligned} \quad (15)$$

We can rewrite the neutrino mass terms as

$$-\mathcal{L}_{mass}^{(\nu)} = \frac{1}{2} \begin{pmatrix} \overline{\nu}_L^C & \overline{\nu}_R & \overline{N}_R \end{pmatrix} M_\nu \begin{pmatrix} \nu_L \\ \nu_R^C \\ N_R^C \end{pmatrix} + \text{H.c.}, \quad (16)$$

where the S_3 flavor symmetry constrains the neutrino mass matrix to be of the form:

$$M_\nu = \begin{pmatrix} 0_{3 \times 3} & i\varepsilon v_\rho & F v_\eta \\ i\varepsilon^T v_\rho & 0_{3 \times 3} & G v_\chi \\ F^T v_\eta & G^T v_\chi & M_R \end{pmatrix}, \quad (17)$$

with

$$\varepsilon^* = \frac{(h_\rho - h_\rho^T) (w_\chi v_\eta \sin \varphi_\chi - w_\eta v_\chi \sin \varphi_\eta)}{\sqrt{2} \Lambda^2} = \begin{pmatrix} 0 & b_3 & b_2 \\ -b_3 & 0 & b_1 \\ -b_2 & -b_1 & 0 \end{pmatrix}, \quad (18)$$

and the submatrices are defined by

$$F = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & a_2 & 0 \\ 0 & a_3 & 0 \end{pmatrix}, \quad G = \begin{pmatrix} 0 & 0 & a_1 \\ 0 & 0 & a_2 \\ 0 & 0 & a_3 \end{pmatrix}, \quad M_R = \begin{pmatrix} \kappa m_N & 0 & 0 \\ 0 & m_N & 0 \\ 0 & 0 & m_N \end{pmatrix}, \quad (19)$$

where $a_j = h_{\Phi j}^{(L)}/\sqrt{2}$ for $j = 1, 2, 3$.

The ansatz for the matrices F , G and M_R follow from the fermion assignments into S_3 irreducible representations, in particular the S_3 discrete symmetry constrains the heavy Majorana neutrino mass matrix M_R to be diagonal.

1. Diagonalization of the mass matrix

Here, for simplicity we assume a scenario corresponding to a double seesaw mechanism [21] where the heavy Majorana neutrino masses and the VEV's satisfy the hierarchy

$$(M_R)_{ll} \gg v_\chi \gg v_\rho, v_\eta \gg w_\chi, w_\eta, \quad l = 1, 2, 3. \quad (20)$$

Resulting from this double seesaw mechanism we have three different mass scales for the neutrinos: very light active neutrinos $\nu_l^{(1)}$, light $\nu_l^{(2)}$ and very heavy sterile neutrinos $\nu_l^{(3)}$ ($l = 1, 2, 3$). As shown in detail in Appendix B, their corresponding mass matrices satisfy the relations:

$$M_\nu^{(1)} \left(M_\nu^{(1)} \right)^T = x v_\eta^2 \left(A - \frac{v_\rho^2}{x v_\eta^2} \varepsilon \varepsilon^T \right), \quad (21)$$

$$M_\nu^{(2)} \left(M_\nu^{(2)} \right)^T = z v_\chi^2 \left(A - \frac{v_\rho^2}{z v_\chi^2} \varepsilon^T \varepsilon \right), \quad (22)$$

$$M_\nu^{(3)} \left(M_\nu^{(3)} \right)^T = \begin{pmatrix} \kappa^2 m_N^2 & 0 & 0 \\ 0 & m_N^2 & 0 \\ 0 & 0 & m_N^2 \end{pmatrix}, \quad (23)$$

with

$$x = (a_1^2 + a_2^2 + a_3^2) \frac{v_\eta^2}{m_N^2}, \quad z = (a_1^2 + a_2^2 + a_3^2) \frac{v_\chi^2}{m_N^2}, \quad (24)$$

where we assumed that the elements of the neutrino mass matrix of Eq. (B1) are real. Moreover, the active light neutrino mass matrix satisfies

$$M_\nu^{(1)} \left(M_\nu^{(1)} \right)^T \simeq x v_\eta^2 \begin{pmatrix} a_1^2 + d_1^2 + d_2^2 & a_1 a_2 + d_1 d_2 & a_1 a_3 - d_1 d_3 \\ a_1 a_2 + d_1 d_2 & a_2^2 + d_1^2 + d_3^2 & a_2 a_3 + d_2 d_3 \\ a_1 a_3 - d_1 d_3 & a_2 a_3 + d_2 d_3 & a_3^2 + d_1^2 + d_2^2 \end{pmatrix}, \quad (25)$$

where

$$d_j = i \frac{v_\rho}{\sqrt{x} v_\eta} b_j, \quad j = 1, 2, 3, \quad (26)$$

and b_j are purely imaginary.

The squared light neutrino mass matrix $M_\nu^{(1)} \left(M_\nu^{(1)} \right)^T$ is diagonalized by a rotation matrix R_ν , according to:

$$R_\nu^T M_\nu^{(1)} \left(M_\nu^{(1)} \right)^T R_\nu = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}, \quad R_\nu = \begin{pmatrix} -\cos \xi_1 \sin \xi_2 & -\sin \xi_1 & \cos \xi_1 \cos \xi_2 \\ \cos \xi_2 & 0 & \sin \xi_2 \\ \sin \xi_1 \sin \xi_2 & \cos \xi_1 & \cos \xi_2 \sin \xi_1 \end{pmatrix}, \quad (27)$$

where

$$\tan \xi_1 \simeq \frac{a_3}{a_1}, \quad \tan 2\xi_2 \simeq \frac{2a_2\sqrt{a_1^2 + a_3^2}}{(a_1^2 + a_3^2 - a_2^2)}. \quad (28)$$

Here we have also assumed that:

$$\sigma_j = \frac{d_j}{a_j} = \frac{\sqrt{2}v_\rho b_j}{\sqrt{x}v_\eta h_{\Phi j}^{(L)}} = \sigma, \quad j = 1, 2, 3. \quad (29)$$

The squared light neutrino masses are given by

$$\begin{aligned} m_1^2 &\simeq \sigma^2 \left[a_1^2 + a_2^2 + a_3^2 - \frac{4a_2^2(a_1^2 + a_3^2)}{(a_1^2 + a_2^2 + a_3^2)} \right] x v_\eta^2, \\ m_2^2 &\simeq \sigma^2 (a_1^2 + a_2^2 + a_3^2) x v_\eta^2, \\ m_3^2 &\simeq (a_1^2 + a_2^2 + a_3^2) x v_\eta^2. \end{aligned} \quad (30)$$

We thus predict a normal hierarchy neutrino mass spectrum, with neutrino mass squared splittings

$$\Delta m_{21}^2 \simeq \frac{4\sigma^2 a_2^2 (a_1^2 + a_3^2) x v_\eta^2}{(a_1^2 + a_2^2 + a_3^2)}, \quad \Delta m_{31}^2 \simeq (a_1^2 + a_2^2 + a_3^2) x v_\eta^2. \quad (31)$$

Notice that both the six-dimensional Yukawa term in Eq. (6) as well as the nonvanishing vacuum expectation values in the first and third components of the χ and η triplets, respectively, are crucial to get a nonzero solar neutrino mass squared splitting Δm_{21}^2 since they correspond to $\sigma \neq 0$. Furthermore, the hierarchy $\Delta m_{21}^2 \ll \Delta m_{31}^2$ can be explained as a consequence of the subleading contribution to the neutrino mass matrix arising from the six dimensional Yukawa term in Eq. (6) proportional to b_j .

The orders of magnitude of the SM particles and new physics give the initial constraints $v_\chi \gtrsim 1$ TeV and $v_\eta^2 + v_\rho^2 = v^2$. We can choose to set the Yukawa couplings $h_{\Phi 1}^{(L)} \sim h_{\Phi 2}^{(L)} \sim h_{\Phi 3}^{(L)}$, following the definition (19) implies $a_j \sim a$ ($j = 1, 2, 3$). We also assume that $b_j \sim b$ ($j = 1, 2, 3$), which means that the elements of the Yukawa matrix h_ρ are of the same order. From Eq. (24), $x \sim a^2 \frac{v_\eta^2}{m_N^2}$ and in first approximation $\Delta m_{31}^2 \sim \frac{a^4 v_\eta^4}{m_N^2}$ [see Eq. (31)]. Therefore, in order to get the right order of magnitude of the atmospheric neutrino mass squared splitting Δm_{31}^2 , we need the heavy neutrinos $N_R^{2,3}$ to have mass $m_N \sim 10^{14} a^2$ GeV. In addition, we also get the estimate $\frac{\Delta m_{31}^2}{\Delta m_{21}^2} \sim \frac{1-\sigma^2}{\sigma^2}$, and since the experimental data on neutrino oscillations implies $\frac{\Delta m_{31}^2}{\Delta m_{21}^2} \sim 30$, this gives $\sigma \sim 10^{-1}$, which results, according to Eq. (29), in $d \sim 10^{-1} a$. From Eq. (31), we get that our estimate $d \sim 10^{-1} a$ yields $a^2 x \sim 10^{-25}$, which implies, according to Eq. (26), that $|b| \sim 10^{-14}$. Furthermore, from Eq. (30) we get for the light active neutrino masses the estimates $m_1 \sim m_2 \sim 6$ meV and $m_3 \sim 30$ meV, which corresponds to a quasidegenerate normal hierarchy neutrino mass spectrum. Besides that, we get that the heaviest sterile neutrino has a mass of about $M_3 \sim 3$ keV. Assuming $|(h_\rho)_{jl}| \sim 1$ ($j, l = 1, 2, 3$), taking into account $|b| \sim 10^{-14}$ and using Eq. (18) and considering $v_\chi \sim 1$ TeV and $w_\chi \sim w_\eta \sim 1$ GeV we get for the cutoff of our model the estimate

$$\Lambda \sim 10^4 - 10^5 \text{ TeV}. \quad (32)$$

B. Charged leptons

Regarding the charged leptons, we assume that the corresponding mass matrix is that one given by the Fukuyama-Nishiura ansatz [27], as follows:

$$M_l = \frac{v_\rho}{\sqrt{2}} h_{\rho e}^{(L)} = \frac{v_\rho}{\sqrt{2}} \begin{pmatrix} 0 & h_1 e^{i\gamma} & h_1 e^{i\gamma} \\ h_1 e^{-i\gamma} & h_2 & h_3 \\ h_1 e^{-i\gamma} & h_3 & h_2 \end{pmatrix} = P_l \widetilde{M}_l P_l^\dagger, \quad (33)$$

where

$$\widetilde{M}_l = \frac{v_\rho}{\sqrt{2}} \begin{pmatrix} 0 & h_1 & h_1 \\ h_1 & h_2 & h_3 \\ h_1 & h_3 & h_2 \end{pmatrix}, \quad P_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\gamma} & 0 \\ 0 & 0 & e^{-i\gamma} \end{pmatrix}. \quad (34)$$

Then, the charged lepton masses are given by

$$-m_e = \frac{v_\rho}{2\sqrt{2}} \left(h_2 + h_3 - \sqrt{(h_2 - h_3)^2 + 8h_1^2} \right), \quad (35)$$

$$m_\mu = \frac{v_\rho}{2\sqrt{2}} \left(h_2 + h_3 + \sqrt{(h_2 - h_3)^2 + 8h_1^2} \right), \quad (36)$$

$$m_\tau = \frac{v_\rho}{\sqrt{2}} (h_2 - h_3), \quad (37)$$

and the mass matrix \widetilde{M}_l is diagonalized by a rotation matrix \widetilde{R}_l according to

$$\widetilde{R}_l^T \widetilde{M}_l \widetilde{R}_l = \begin{pmatrix} -m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad \widetilde{R}_l = \begin{pmatrix} c_l & s_l & 0 \\ -\frac{s_l}{\sqrt{2}} & \frac{c_l}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_l}{\sqrt{2}} & \frac{c_l}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (38)$$

with

$$c_l = \cos \theta_l = \sqrt{\frac{m_\mu}{m_\mu + m_e}}, \quad s_l = \sin \theta_l = \sqrt{\frac{m_e}{m_\mu + m_e}}. \quad (39)$$

Putting it all together, the charged lepton mass matrix M_l is diagonalized by a rotation matrix R_l according to

$$R_l^\dagger M_l R_l = \begin{pmatrix} -m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad R_l = P_l \widetilde{R}_l. \quad (40)$$

C. Lepton mixing

With the rotation matrices in the charged lepton sector R_l given in Eq. (40), and in the neutrino sector R_ν given in Eq. (27), we find the PMNS leptonic mixing matrix

$$U = \begin{pmatrix} -c_1 s_2 c_l - \frac{1}{\sqrt{2}} c_2 s_l e^{i\gamma} - \frac{1}{\sqrt{2}} s_1 s_2 s_l e^{i\gamma} & -s_1 c_l - \frac{1}{\sqrt{2}} c_1 s_l e^{i\gamma} & c_1 c_2 c_l - \frac{1}{\sqrt{2}} s_2 s_l e^{i\gamma} - \frac{1}{\sqrt{2}} c_2 s_1 s_l e^{i\gamma} \\ \frac{1}{\sqrt{2}} c_2 c_l e^{i\gamma} - c_1 s_2 s_l + \frac{1}{\sqrt{2}} s_1 s_2 c_l e^{i\gamma} & \frac{1}{\sqrt{2}} c_1 c_l e^{i\gamma} - s_1 s_l & c_1 c_2 s_l + \frac{1}{\sqrt{2}} s_2 c_l e^{i\gamma} + \frac{1}{\sqrt{2}} c_2 s_1 c_l e^{i\gamma} \\ \frac{1}{\sqrt{2}} s_1 s_2 e^{i\gamma} - \frac{1}{\sqrt{2}} c_2 e^{i\gamma} & \frac{1}{\sqrt{2}} c_1 e^{i\gamma} & \frac{1}{\sqrt{2}} c_2 s_1 e^{i\gamma} - \frac{1}{\sqrt{2}} s_2 e^{i\gamma} \end{pmatrix}. \quad (41)$$

From the standard parametrization of the lepton mixing matrix, it follows that the lepton mixing angles are

$$\sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \quad (42)$$

$$= \frac{\frac{1}{2} \sin^2 \gamma \sin^2 \theta_l \cos^2 \xi_1 + \frac{1}{4} \left(-\sqrt{2} \cos \gamma \sin \theta_l \cos \xi_1 - 2 \cos \theta_l \sin \xi_1 \right)^2}{1 - \frac{1}{2} \sin^2 \gamma \sin^2 \theta_l (\sin \xi_2 + \sin \xi_1 \cos \xi_2)^2 - \frac{1}{4} \left[2 \cos \theta_l \cos \xi_1 \cos \xi_2 - \sqrt{2} \cos \gamma \sin \theta_l (\sin \xi_2 + \sin \xi_1 \cos \xi_2) \right]^2}, \quad (43)$$

$$\sin^2 \theta_{13} = |U_{e3}|^2 \quad (44)$$

$$= \frac{1}{2} \sin^2 \gamma \sin^2 \theta_l (\sin \xi_2 + \sin \xi_1 \cos \xi_2)^2 + \frac{1}{4} \left[2 \cos \theta_l \cos \xi_1 \cos \xi_2 - \sqrt{2} \cos \gamma \sin \theta_l (\sin \xi_2 + \sin \xi_1 \cos \xi_2) \right]^2, \quad (45)$$

$$\sin^2 \theta_{23} = \frac{|U_{\mu 3}|^2}{1 - |U_{e 3}|^2} \quad (46)$$

$$= \frac{\frac{1}{2} \sin^2 \gamma \cos^2 \theta_l (\sin \xi_2 + \sin \xi_1 \cos \xi_2)^2 + \frac{1}{4} (\sqrt{2} \cos \gamma \cos \theta_l (\sin \xi_2 + \sin \xi_1 \cos \xi_2) + 2 \sin \theta_l \cos \xi_1 \cos \xi_2)^2}{1 - \frac{1}{2} \sin^2 \gamma \sin^2 \theta_l (\sin \xi_2 + \sin \xi_1 \cos \xi_2)^2 - \frac{1}{4} [2 \cos \theta_l \cos \xi_1 \cos \xi_2 - \sqrt{2} \cos \gamma \sin \theta_l (\sin \xi_2 + \sin \xi_1 \cos \xi_2)]^2}. \quad (47)$$

The Jarlskog invariant and the CP violating phase are respectively given by [3, 28, 29]

$$J = \text{Im} (U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*) = -\frac{\sin \gamma \sin 2\theta_l}{32\sqrt{2}} [4 \cos^2 \xi_2 \sin 2\xi_1 - 6 \sin 4\xi_1 \sin^2 \xi_2 + \sin 2\xi_2 (\cos \xi_1 - 5 \cos 3\xi_1)], \quad (48)$$

$$\sin \delta = \frac{(1 - |U_{e3}|^2) J}{|U_{e1} U_{e2} U_{e3} U_{\mu 3} U_{\tau 3}|}. \quad (49)$$

Varying the parameters ξ_1 , ξ_2 and γ we have fitted the $\sin^2 \theta_{ij}$ to the experimental values in Table II for the normal hierarchy neutrino mass spectrum. From Eq. (39), we have

$$\sin 2\theta_l \simeq 2 \sqrt{\frac{m_e}{m_\mu}}. \quad (50)$$

The best fit result is

$$\xi_1 = 212.8^\circ, \quad \xi_2 = 101.7^\circ, \quad \gamma = 66.4^\circ. \quad (51)$$

$$\sin^2 \theta_{12} = 0.32, \quad \sin^2 \theta_{23} = 0.613, \quad \sin^2 \theta_{13} = 0.0246, \quad (52)$$

$$J = -9.81 \times 10^{-3}, \quad \delta = 16^\circ.$$

Comparing the results in (52) with the values in Table II, we see that the mixing parameters $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ are in agreement with the experimental data. We obtain that CP is violated in neutrino oscillations with a Jarlskog invariant of about 10^{-2} . Furthermore, the complex phase γ responsible for CP violation in lepton sector arises from the Yukawa terms for the charged leptons.

Parameter	$\Delta m_{21}^2 (10^{-5} \text{eV}^2)$	$\Delta m_{31}^2 (10^{-3} \text{eV}^2)$	$(\sin^2 \theta_{12})_{\text{exp}}$	$(\sin^2 \theta_{23})_{\text{exp}}$	$(\sin^2 \theta_{13})_{\text{exp}}$
Best fit	7.62	2.55	0.320	0.613	0.0246
1σ range	7.43 – 7.81	2.46 – 2.61	0.303 – 0.336	0.573 – 0.635	0.0218 – 0.0275
2σ range	7.27 – 8.01	2.38 – 2.68	0.29 – 0.35	0.38 – 0.66	0.019 – 0.030
3σ range	7.12 – 8.20	2.31 – 2.74	0.27 – 0.37	0.36 – 0.68	

Table II: Range for experimental values of neutrino mass squared splittings and leptonic mixing parameters taken from Ref. [9] for the case of normal hierarchy.

IV. CONCLUSIONS

We proposed a model based on the group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes S_3$ where lepton masses and mixing can be reproduced. We assumed that the heavy Majorana neutrinos have masses much larger than the TeV scale, so that the hierarchy $m_N \gg v_\chi \gg v_\rho, v_\eta$ is fulfilled, implying that the small active neutrino masses are generated via a double seesaw mechanism. We found that the only $SU(3)_L \otimes U(1)_X \otimes S_3$ invariant nonrenormalizable operators of lowest order that contribute to the neutrino masses are $\frac{1}{\Lambda} \bar{N}_R^1 N_R^C (\Phi \Phi^\dagger)$ and $\frac{1}{\Lambda^2} \bar{L}_L L_L^C \rho (\Phi^\dagger \Phi)$. From these nonrenormalizable terms, $\frac{1}{\Lambda^2} \bar{L}_L L_L^C \rho (\Phi^\dagger \Phi)$ gives a relevant contribution to the neutrino masses since the operator $\frac{1}{\Lambda} \bar{N}_R^1 N_R^C (\Phi \Phi^\dagger)$ gives a subleading contribution to the heavy Majorana neutrino masses.

In this scenario, the spectrum of neutrinos presents very light, light and very heavy masses. Assuming that the heavy Majorana neutrinos have masses of about $m_N \sim 10^{14}$ GeV, we find for the light active neutrino masses the estimates $m_1 \sim m_2 \sim 6$ meV and $m_3 \sim 30$ meV, while for the heaviest sterile neutrino mass we get $M_3 \sim 3$ keV. The model predicts a quasidegenerate normal hierarchy active neutrino mass spectrum and the relation $\Delta m_{21}^2 \ll \Delta m_{31}^2$ can be explained as consequence of the small mass terms arising from the effective six- dimensional operators. We find for the scale of these effective operators the estimate $\Lambda \sim 10^4 - 10^5$ TeV. These effective operators generate a nonvanishing solar neutrino mass squared splitting Δm_{21}^2 , provided that the first and third components of the χ and η triplets, respectively, should have nonvanishing vacuum expectation values, where at least one of them has to be complex. The obtained neutrino mixing parameters are in excellent agreement with the neutrino oscillation experimental data. We find that CP is violated in neutrino oscillations with a Jarlskog invariant of about 10^{-2} . Furthermore, the complex phase responsible for CP violation in the lepton sector has been assumed to come from the Yukawa terms for the charged leptons.

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Appendix A: The product rules for S_3

The S_3 group has three irreducible representations: **1**, **1'** and **2**. Denoting $(x_1, x_2)^T$ and $(y_1, y_2)^T$ as the basis vectors for two S_3 doublets and y a nontrivial S_3 singlet, the multiplication rules of the S_3 group for the case of real representations are given by [19]

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_{\mathbf{2}} = (x_1 y_1 + x_2 y_2)_{\mathbf{1}} + (x_1 y_2 - x_2 y_1)_{\mathbf{1}'} + \begin{pmatrix} x_1 y_2 + x_2 y_1 \\ x_1 y_1 - x_2 y_2 \end{pmatrix}_{\mathbf{2}}, \quad (\text{A1})$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes (y)_{\mathbf{1}'} = \begin{pmatrix} -x_2 y \\ x_1 y \end{pmatrix}_{\mathbf{2}}, \quad (x)_{\mathbf{1}'} \otimes (y)_{\mathbf{1}'} = (xy)_{\mathbf{1}}. \quad (\text{A2})$$

Appendix B: Diagonalization of the neutrino mass matrix

We consider the neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0_{3 \times 3} & i\varepsilon v_\rho & F v_\eta \\ i\varepsilon^T v_\rho & 0_{3 \times 3} & G v_\chi \\ F^T v_\eta & G^T v_\chi & M_R \end{pmatrix} = \begin{pmatrix} \widetilde{M}_\nu & C \\ C^T & M_R \end{pmatrix}, \quad (\text{B1})$$

where the different sub-blocks are given by Eqs. (18)-(19).

For the sake of simplicity we assume that the elements of the neutrino mass matrix of Eq. (B1) are real. Now, in order to block-diagonalize the mass matrix M_ν , we apply the transformation

$$W^T M_\nu W \simeq \begin{pmatrix} \widetilde{M}_\nu - C B^T - B C^T + B M_R B^T & C - B M_R \\ C^T - M_R B^T & M_R + C^T B + B^T C \end{pmatrix},$$

with

$$W = \begin{pmatrix} 1_{6 \times 6} & B \\ -B^T & 1_{3 \times 3} \end{pmatrix}. \quad (\text{B2})$$

Using the method of recursive expansion of Ref. [30], we find that the block diagonalization condition leads to

$$B \simeq C M_R^{-1}, \quad B^T \simeq M_R^{-1} C^T. \quad (\text{B3})$$

Then, it follows that:

$$W^T M_\nu W \simeq \begin{pmatrix} \widetilde{M}_\nu - C M_R^{-1} C^T & 0_{6 \times 3} \\ 0_{3 \times 6} & M_R \end{pmatrix}, \quad C M_R^{-1} C^T = \begin{pmatrix} F M_R^{-1} F^T v_\eta^2 & F M_R^{-1} G^T v_\eta v_\chi \\ G M_R^{-1} F^T v_\eta v_\chi & G M_R^{-1} G^T v_\chi^2 \end{pmatrix}. \quad (\text{B4})$$

The previous relations imply:

$$W^T M_\nu W \simeq \begin{pmatrix} -\frac{v_\eta^2}{m_N} A & i\varepsilon v_\rho & 0_{3 \times 3} \\ i\varepsilon^T v_\rho & -\frac{v_\chi^2}{m_N} A & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & M_R \end{pmatrix} = \begin{pmatrix} S_\nu & 0_{6 \times 3} \\ 0_{3 \times 6} & M_R \end{pmatrix}, \quad (\text{B5})$$

where

$$S_\nu = \begin{pmatrix} -\frac{v_\eta^2}{m_N} A & i\varepsilon v_\rho \\ i\varepsilon^T v_\rho & -\frac{v_\chi^2}{m_N} A \end{pmatrix}. \quad (\text{B6})$$

Moreover, the matrix $S_\nu S_\nu^T$ gives

$$S_\nu S_\nu^T = \begin{pmatrix} X & Y \\ Y^T & Z \end{pmatrix}, \quad (\text{B7})$$

where

$$X = x v_\eta^2 A - v_\rho^2 \varepsilon \varepsilon^T, \quad Z = z v_\chi^2 A - v_\rho^2 \varepsilon^T \varepsilon, \quad Y = i y v_\rho v_\chi \varepsilon A, \quad (\text{B8})$$

with

$$\begin{aligned} x &= (a_1^2 + a_2^2 + a_3^2) \frac{v_\eta^2}{m_N^2}, & y &= -\frac{v_\chi}{m_N}, \\ z &= (a_1^2 + a_2^2 + a_3^2) \frac{v_\chi^2}{m_N^2}. \end{aligned} \quad (\text{B9})$$

Furthermore, the following hierarchy is fulfilled:

$$|Y_{ij}| \ll |X_{ij}| \ll |Z_{ij}|. \quad (\text{B10})$$

Now, in order to block-diagonalize the matrix $S_\nu S_\nu^T$, we apply the transformation

$$P^T S_\nu S_\nu^T P \simeq \begin{pmatrix} X - Y K^T - K Y^T + K Z K^T & Y - K Z \\ Y^T - Z K^T & Z + Y^T K + K^T Y + K^T X K \end{pmatrix},$$

with

$$P = \begin{pmatrix} 1_{3 \times 3} & K \\ -K^T & 1_{3 \times 3} \end{pmatrix}. \quad (\text{B11})$$

The block diagonalization condition leads to the relations

$$K \simeq Y Z^{-1}, \quad K^T = Z^{-1} Y^T. \quad (\text{B12})$$

Therefore, we obtain

$$P^T S_\nu S_\nu^T P \simeq \begin{pmatrix} M_\nu^{(1)} \left(M_\nu^{(1)} \right)^T & 0_{3 \times 3} \\ 0_{3 \times 3} & M_\nu^{(2)} \left(M_\nu^{(2)} \right)^T \end{pmatrix}, \quad (\text{B13})$$

where

$$M_\nu^{(1)} \left(M_\nu^{(1)} \right)^T \simeq x v_\eta^2 \left(A - \frac{v_\rho^2}{x v_\eta^2} \varepsilon \varepsilon^T \right), \quad (\text{B14})$$

$$M_\nu^{(2)} \left(M_\nu^{(2)} \right)^T \simeq z v_\chi^2 \left(A - \frac{v_\rho^2}{z v_\chi^2} \varepsilon^T \varepsilon \right). \quad (\text{B15})$$

Notice that $M_\nu^{(1)} \left(M_\nu^{(1)} \right)^T$ corresponds to the squared active light neutrino mass matrix. Moreover, Eq. (B14) can be rewritten as follows:

$$M_\nu^{(1)} \left(M_\nu^{(1)} \right)^T \simeq x v_\eta^2 \begin{pmatrix} a_1^2 + d^2 - d_1^2 & a_1 a_2 + d_1 d_2 & a_1 a_3 - d_1 d_3 \\ a_1 a_2 + d_1 d_2 & a_2^2 + d^2 - d_2^2 & a_2 a_3 + d_2 d_3 \\ a_1 a_3 - d_1 d_3 & a_2 a_3 + d_2 d_3 & a_3^2 + d^2 - d_3^2 \end{pmatrix}, \quad (\text{B16})$$

where

$$\tan 2\xi_1 = \frac{2(a_1 a_3 - d_1 d_3)}{a_1^2 - a_3^2 - d_1^2 + d_3^2}, \quad \tan 2\xi_2 = \frac{2a_2(1 + \sigma^2) \sqrt{a_1^2 + a_3^2}}{(1 - \sigma^2)(a_1^2 + a_3^2 - a_2^2)}, \quad (\text{B17})$$

so that the squared neutrino masses are given by

$$\begin{aligned} m_1^2 &= \frac{x v_\eta^2}{2} (1 + \sigma^2) (a_1^2 + a_2^2 + a_3^2) - \frac{x v_\eta^2}{2} \sqrt{(1 + \sigma^4) (a_1^2 + a_2^2 + a_3^2)^2 - 2\sigma^2 [(a_1^2 + a_3^2 + a_2^2)^2 - 8a_2^2 (a_1^2 + a_3^2)]}, \\ m_2^2 &= x \sigma^2 (a_1^2 + a_2^2 + a_3^2) v_\eta^2, \\ m_3^2 &= \frac{x v_\eta^2}{2} (1 + \sigma^2) (a_1^2 + a_2^2 + a_3^2) + \frac{x v_\eta^2}{2} \sqrt{(1 + \sigma^4) (a_1^2 + a_2^2 + a_3^2)^2 - 2\sigma^2 [(a_1^2 + a_3^2 + a_2^2)^2 - 8a_2^2 (a_1^2 + a_3^2)]}. \end{aligned} \quad (\text{B18})$$

Thus, the squared light neutrino mass matrix $M_\nu^{(1)} \left(M_\nu^{(1)} \right)^T$ is diagonalized by a rotation matrix R_ν , according to

$$R_\nu^T M_\nu^{(1)} \left(M_\nu^{(1)} \right)^T R_\nu = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}, \quad (\text{B19})$$

where

$$R_\nu = \begin{pmatrix} -\cos \xi_1 \sin \xi_2 & -\sin \xi_1 & \cos \xi_1 \cos \xi_2 \\ \cos \xi_2 & 0 & \sin \xi_2 \\ \sin \xi_1 \sin \xi_2 & \cos \xi_1 & \cos \xi_2 \sin \xi_1 \end{pmatrix}. \quad (\text{B20})$$

Similarly, the sterile neutrino mass matrix satisfies

$$M_\nu^{(2)} \left(M_\nu^{(2)} \right)^T \simeq z v_\chi^2 \begin{pmatrix} a_1^2 + p_2^2 + p_3^2 & a_1 a_2 + p_1 p_2 & a_1 a_3 - p_1 p_3 \\ a_1 a_2 + p_1 p_2 & a_2^2 + p_1^2 + p_3^2 & a_2 a_3 + p_2 p_3 \\ a_1 a_3 - p_1 p_3 & a_2 a_3 + p_2 p_3 & a_3^2 + p_1^2 + p_2^2 \end{pmatrix}, \quad (\text{B21})$$

where

$$p_j = i \frac{v_\rho}{\sqrt{z} v_\chi} b_j, \quad j = 1, 2, 3, \quad (\text{B22})$$

and b_j are purely imaginary.

Furthermore, following the same procedure used for the light active neutrinos, we get that the squared sterile neutrino masses are given by

$$\begin{aligned} M_1^2 &\simeq \theta^2 \left[a_1^2 + a_2^2 + a_3^2 - \frac{4a_2^2(a_1^2 + a_3^2)}{(a_1^2 + a_2^2 + a_3^2)} \right] z v_\chi^2, \\ M_2^2 &\simeq \theta^2 (a_1^2 + a_2^2 + a_3^2) z v_\chi^2, \\ M_3^2 &\simeq (a_1^2 + a_2^2 + a_3^2) z v_\chi^2. \end{aligned} \quad (\text{B23})$$

where:

$$\theta_j = \frac{p_j}{a_j} = \frac{\sqrt{2} v_\rho b_j}{\sqrt{z} v_\chi h_{\Phi j}^{(L)}} = \theta, \quad j = 1, 2, 3. \quad (\text{B24})$$

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